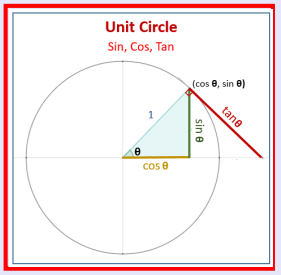
Math 241

Winter 2023

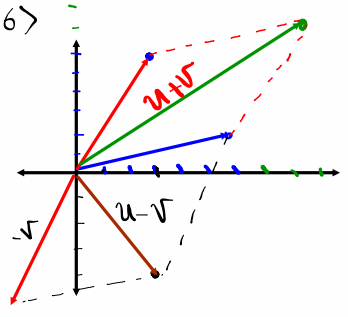
Lecture 16



Feb 19-8:47 AM

Given $u = \langle 6, 2 \rangle$, $v = \langle 3, 6 \rangle$

1) Draw u & v.



2) Find $u + v$, and draw it.

$u+v = \langle 6+3, 2+6 \rangle = \langle 9, 8 \rangle$

3) Find $u - v$ and draw it.

$u-v = \langle 6-3, 2-6 \rangle = \langle 3, -4 \rangle$

4) Find the angle between two vectors u & v.

$$\cos\theta = \frac{u \cdot v}{|u||v|} \Rightarrow \theta = \cos^{-1}\left(\frac{u \cdot v}{|u||v|}\right)$$

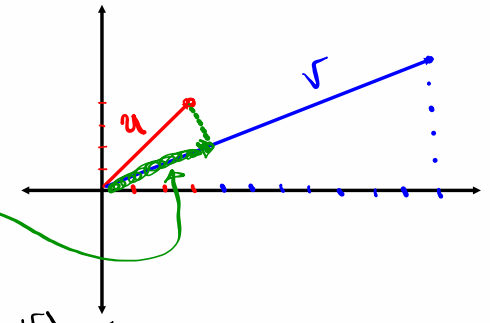
$$= \cos^{-1}\left(\frac{30}{\sqrt{40}\sqrt{45}}\right)$$

$$\approx \cos^{-1}(.707) \approx \boxed{45^\circ}$$

Jan 31-7:06 AM

Given $u = \langle 3, 4 \rangle$, $v = \langle 12, 5 \rangle$

1) Draw u & v



2) Draw $\text{Proj}_v u$

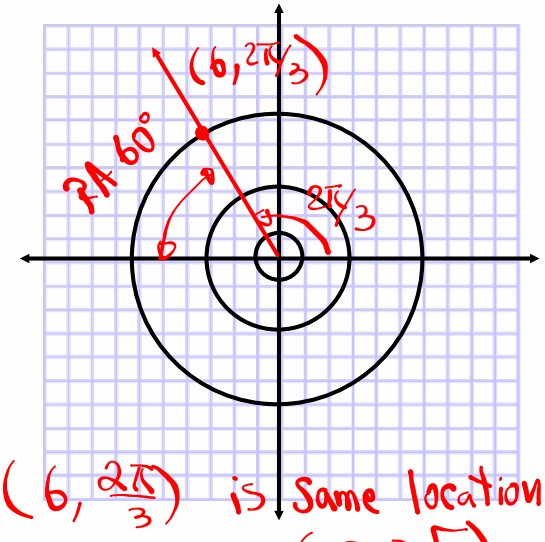
3) Find $\text{Proj}_v u = \left(\frac{u \cdot v}{|v|}\right) \cdot \frac{1}{|v|} v$

$= \frac{u \cdot v}{|v|^2} v = \frac{56}{169} \langle 12, 5 \rangle$

$= \left\langle \frac{672}{169}, \frac{280}{169} \right\rangle$

Jan 31-7:20 AM

Plot the Polar Point $(6, \frac{2\pi}{3})$, then find the rectangular coordinates for it.



$(6, \frac{2\pi}{3})$ is same location as $(-3, 3\sqrt{3})$

$(6, \frac{2\pi}{3})$

$r = 6$ $\quad \theta = \frac{2\pi}{3} = 120°$

$x = r\cos\theta$

$= 6\cos\frac{2\pi}{3} = 6\left(\frac{-1}{2}\right) = -3$

$y = r\sin\theta$

$= 6\sin\frac{2\pi}{3} = 6\left(\frac{\sqrt{3}}{2}\right) = 3\sqrt{3}$

Jan 31-7:28 AM

Plot the rectangular coordinate point $(-\sqrt{8}, -\sqrt{8})$ then find the polar coordinates with $r>0$, and $0 \le \theta \le 2\pi$.

$(-\sqrt{8}, -\sqrt{8})$

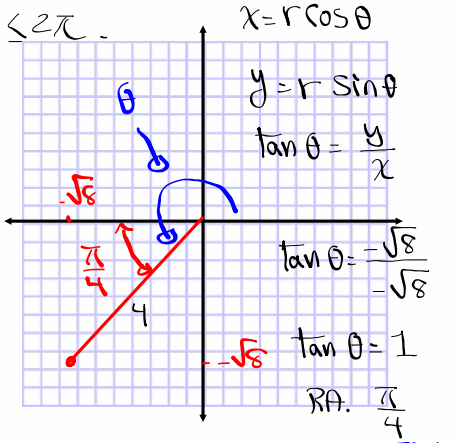
$x = -\sqrt{8} \quad y = -\sqrt{8}$

$r^2 = x^2 + y^2$

$r^2 = (-\sqrt{8})^2 + (-\sqrt{8})^2$

$r^2 = 8 + 8 = 16$

$\boxed{r = 4}$



$x = r\cos\theta$

$y = r\sin\theta$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{-\sqrt{8}}{-\sqrt{8}}$

$\tan\theta = 1$

RA. $\frac{\pi}{4}$

$\theta = \pi + \pi/4 = 5\pi/4$

$(-\sqrt{8}, -\sqrt{8})$ is on same location as $(4, \frac{5\pi}{4})$

Jan 31-7:34 AM

$x^2 + y^2 = 16$ Rectangular equation

$r^2 = 16$

$r = 4$ Polar equation

$x = y$ Rectangular

Divide by x

$1 = \frac{y}{x}$, $x \neq 0$ $\quad 1 = \tan\theta$ Polar eqn.

$\theta = \frac{\pi}{4} + n\pi$

Jan 31-7:41 AM

$y = 4$ Rectangular

$r\sin\theta = 4$ Polar

$x = -2$ Rectangular

$r\cos\theta = -2$ Polar

Convert $x^2 - y^2 = 1$ to a polar eqn.

$(r\cos\theta)^2 - (r\sin\theta)^2 = 1$

$r^2\cos^2\theta - r^2\sin^2\theta = 1$

$r^2(\cos^2\theta - \sin^2\theta) = 1$

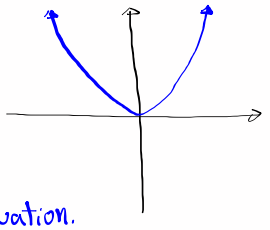
$r^2 \cdot \cos 2\theta = 1$

$r^2 = \frac{1}{\cos 2\theta}$ $r^2 = \sec 2\theta$

Jan 31-7:44 AM

Given $y = x^2$

1) Graph it



2) Convert to a polar equation.

$y = x^2$

$r\sin\theta = (r\cos\theta)^2$

$r\sin\theta = r^2\cos^2\theta$

Divide by r (if $r \neq 0$)

$\sin\theta = r\cos^2\theta$

Divide by $\cos^2\theta$ (if $\cos\theta \neq 0$)

$\frac{\sin\theta}{\cos^2\theta} = r$

$r = \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$

$r = \tan\theta \cdot \sec\theta$

Jan 31-7:48 AM

Convert $r = 5$ to rectangular.

$r^2 = 25$

$\rightarrow x^2 + y^2 = 25$ ✓

Convert $r = \frac{6}{\cos\theta}$ to rectangular.

Hint: Multiply by LCD to clear fraction.

$r\cos\theta = 6$

$x = 6$ ✓

Jan 31-7:53 AM

Convert to rectangular equations:

1) $r = -4\csc\theta$ $\quad r = \frac{-4}{\sin\theta}$ $\quad r\sin\theta = -4$

$y = -4$

2) $r = 6\cos\theta$

Hint: Multiply both sides by r

$r \cdot r = 6r\cos\theta$

$r^2 = 6r\cos\theta$

$\boxed{x^2 + y^2 = 6x}$

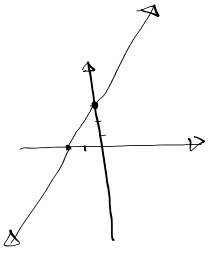
3) $r = \frac{6}{2\sin\theta - 3\cos\theta}$ Hint: Cross-Multiply

$r(2\sin\theta - 3\cos\theta) = 6$

$2r\sin\theta - 3r\cos\theta = 6$

$2y - 3x = 6$

x	y
0	3
-2	0



Jan 31-7:57 AM

Convert to rectangular equation:

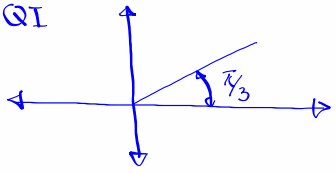
$r^2 = \tan\theta$

$x^2 + y^2 = \frac{y}{x}$

$\sec\theta = 2 \Rightarrow \cos\theta = \frac{1}{2}$ QI & QIV

R.A. $\pi/3$

QI

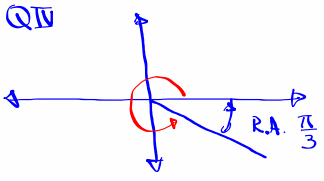


$\tan\theta = \frac{y}{x}$

$\tan\frac{\pi}{3} = \frac{y}{x}$

$\sqrt{3} = \frac{y}{x} \Rightarrow \boxed{y = \sqrt{3}x}$

QIV



$\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$\tan\theta = \frac{y}{x}$

$\tan\frac{5\pi}{3} = \frac{y}{x}$

$-\sqrt{3} = \frac{y}{x}$

$\boxed{y = -\sqrt{3}x}$

$\sec\theta = 2$ Polar

$y = \pm\sqrt{3}x$ Rectangular

Jan 31-8:05 AM

Convert $r^2 = \sin 2\theta$ to rectangular eqn.

$r^2 = 2\sin\theta\cos\theta$

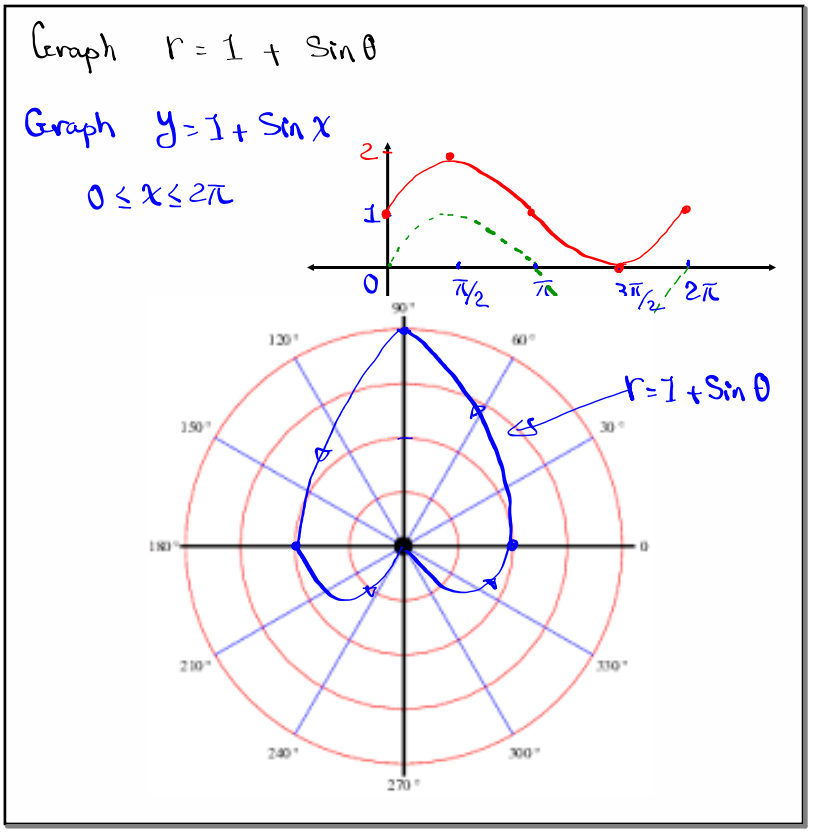
multiply both sides by r^2

$r^2 r^2 = r^2 2\sin\theta\cos\theta$

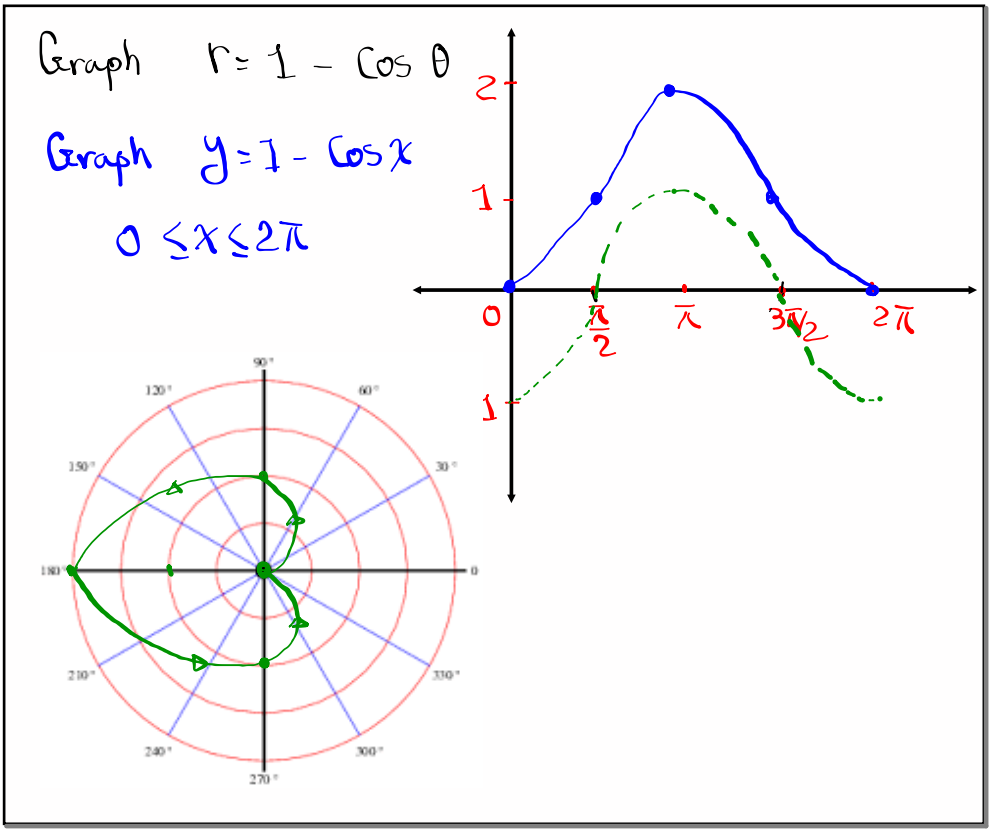
$(r^2)^2 = 2 r\sin\theta \cdot r\cos\theta$

$\boxed{(x^2+y^2)^2 = 2yx}$

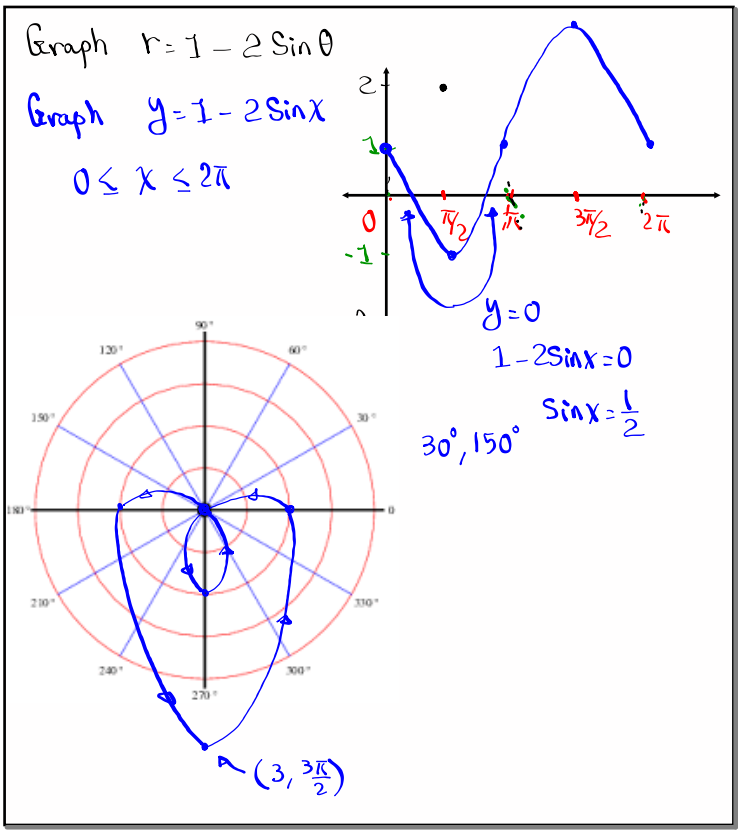
Jan 31-8:13 AM



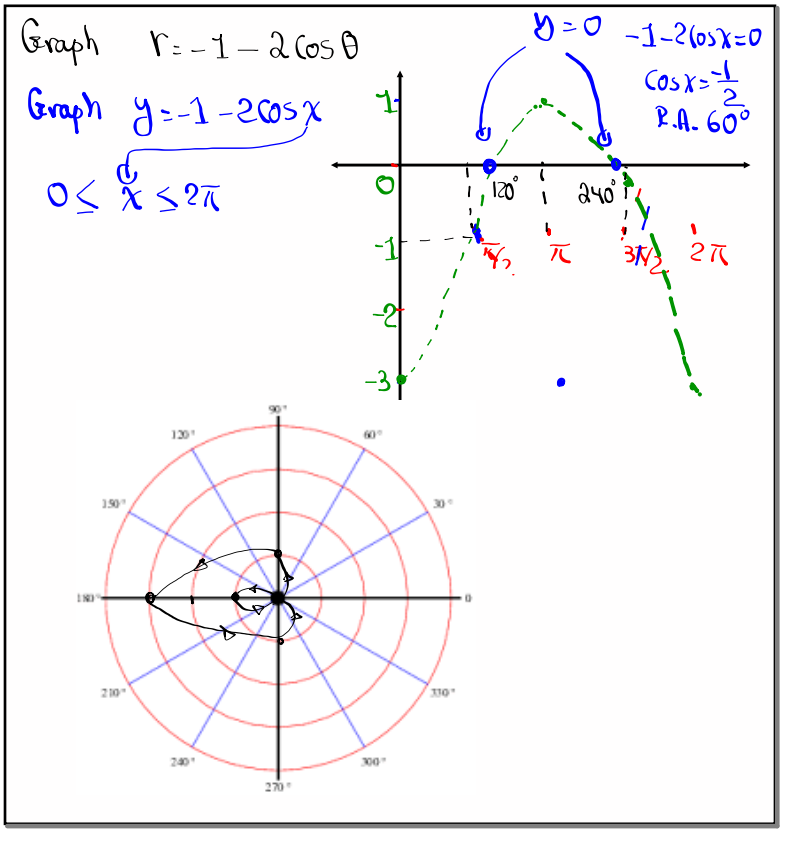
Jan 31-9:00 AM



Jan 31-9:06 AM



Jan 31-9:13 AM



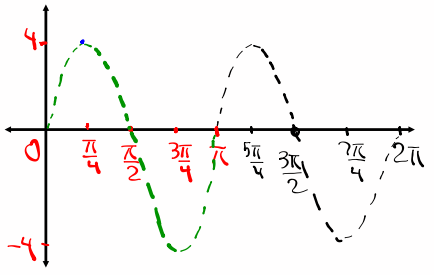
Jan 31-9:22 AM

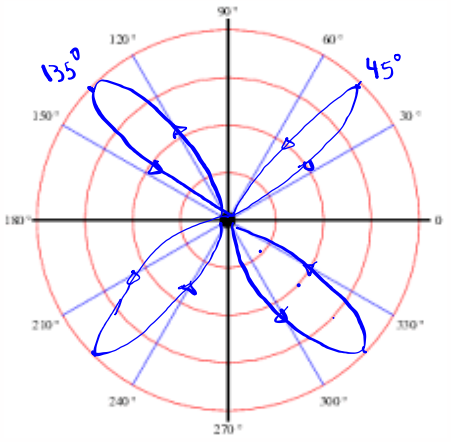
Graph $r = 4 \sin 2\theta$

Graph $y = 4 \sin 2x$

$0 \leq 2x \leq 2\pi$

$0 \leq x \leq \pi$





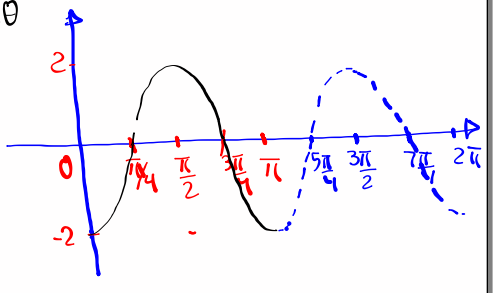
Jan 31-9:33 AM

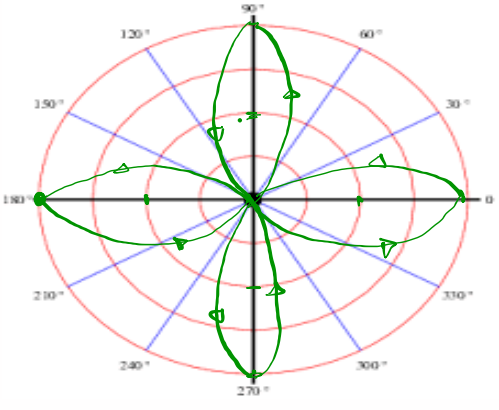
Graph $r = -2 \cos 2\theta$

$y = -2 \cos 2x$

$0 \leq 2x \leq 2\pi$

$0 \leq x \leq \pi$





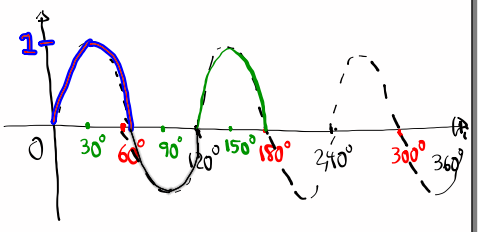
Jan 31-9:40 AM

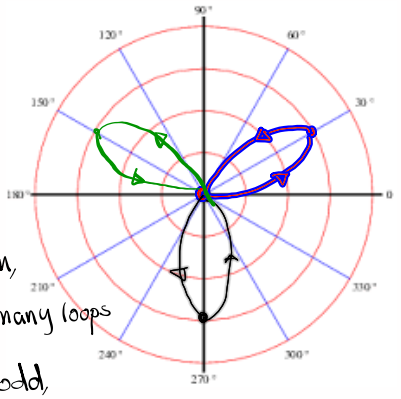
Graph $r = \sin 3\theta$

Graph $y = \sin 3x$

$0° \le 3x \le 360°$

$0° \le x \le 120°$





$\sin n\theta$

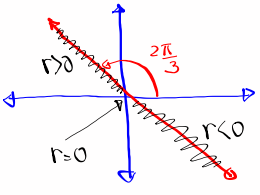
$\cos n\theta$

if n is even, twice as many loops

if n is odd, get n loops

Jan 31-9:47 AM

Graph $\theta = \frac{2\pi}{3}$ $\quad \frac{2\pi}{3} = 2(60°) = 120°$



Graph $r = 8\cos\theta$

$r^2 = 8r\cos\theta$

$x^2 + y^2 = 8x$

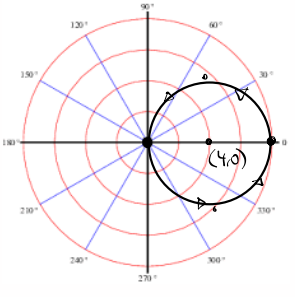
$x^2 - 8x + 16 + y^2 = 0 + 16$

$(x-4)^2 + (y-0)^2 = 16$

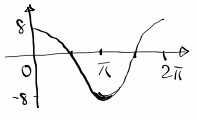
Circle

center $(4,0)$

radius 4



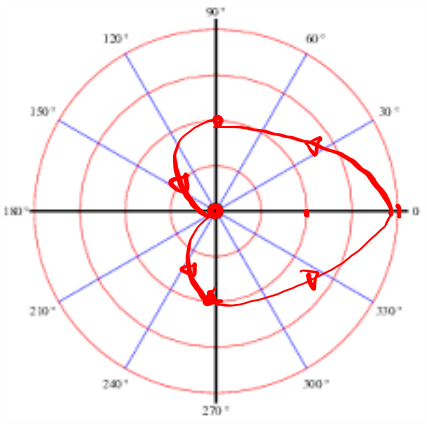
$y = 8\cos x$

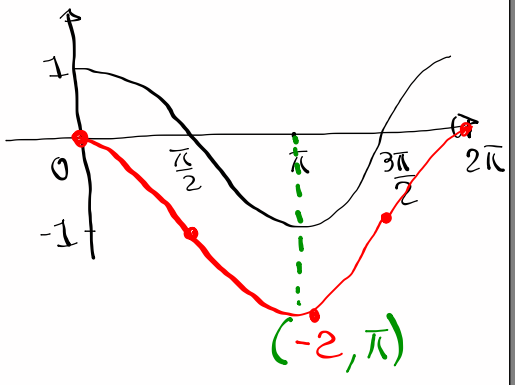


Jan 31-10:31 AM

Graph $r = \cos\theta - 1$

$r = -1 + \cos\theta$



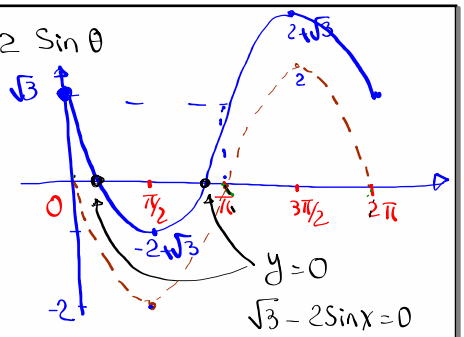


Jan 31-10:38 AM

Graph $r = \sqrt{3} - 2\sin\theta$

$y = \sqrt{3} - 2\sin x$

$0 \le x \le 2\pi$



$y = 0$

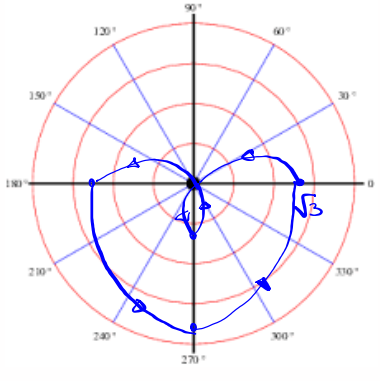
$\sqrt{3} - 2\sin x = 0$

$\sin x = \frac{\sqrt{3}}{2}$

R.A. 60°

QI $x = 60^\circ$

QII $x = 180^\circ - 60^\circ = 120^\circ$



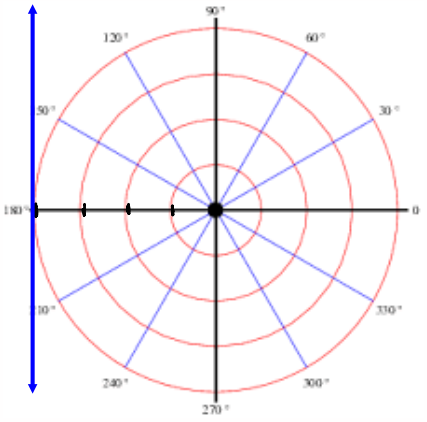
Jan 31-10:43 AM

Graph $r = -4 \sec\theta$

$r = \frac{-4}{\cos\theta}$

$r\cos\theta = -4$

$x = -4$



Jan 31-10:51 AM

Graph $r^2 = \cos 2\theta$

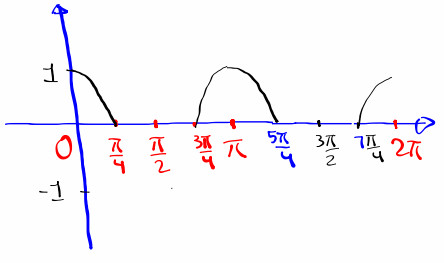
$r^2 \geq 0$

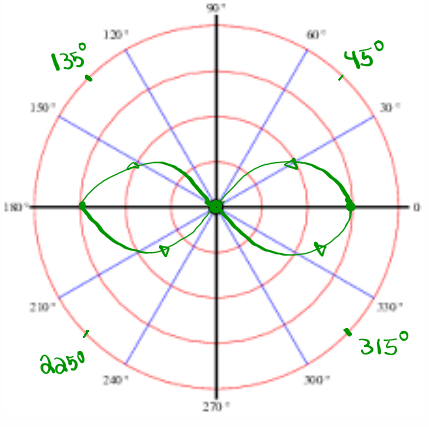
Let's graph

$y = \cos 2x$

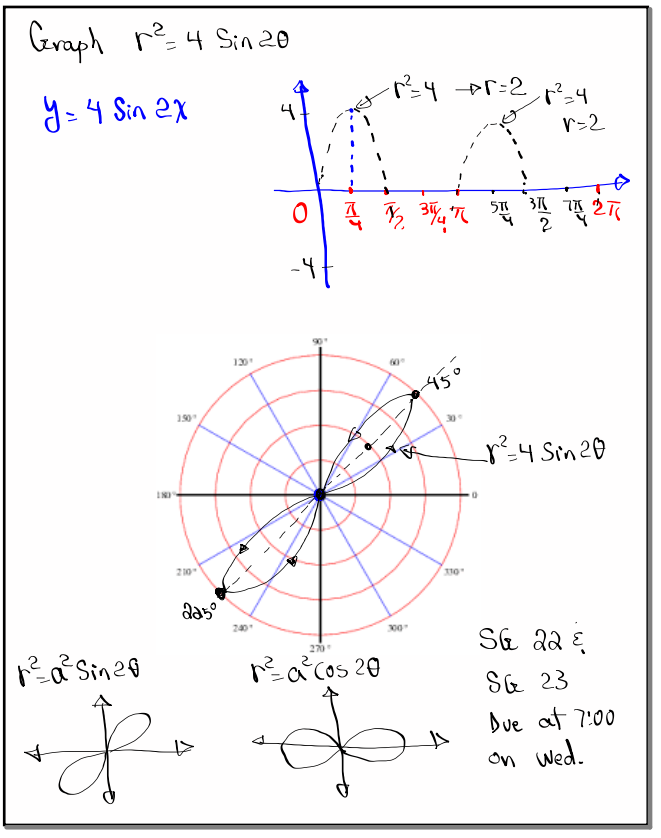
$0 \leq 2x \leq 2\pi$

$0 \leq x \leq \pi$





Jan 31-10:54 AM



Jan 31-11:03 AM